

Sequences and Series

Sequence:

1, 2, 3, 5, 8, 13, 21, 34, ...

term \rightarrow

$$a_1 = 1$$
$$a_2 = 2$$
$$a_3 = 3$$

$$a_8 = 34$$

x $y = 2x + 1$

1	3	$a_1 = 3$
2	5	
3	7	
4	9	$a_4 = 9$
5	11	
⋮		
n		$a_n = 2n + 1$

3, 5, 7, 9, 11

Write the first four terms of the sequence given by:

$$a_n = n^2 + 1$$

$$a_1 = 2$$

$$a_2 = 5$$

$$a_3 = 10$$

$$a_4 = 17$$

? Finding a pattern ?

Write an expression for the apparent n th term of each sequence:

a) 1, 3, 5, 7,

1	2	3	4	...	n
1	3	5	7		$2n-1$
✓	✓	✓			
2	2	2			

b) 0, 3, 8, 15, 24, ...

1	2	3	4	5	n
0	3	8	15	24	n^2-1
	✓	✓	✓	✓	
	3	5	7	9	
		✓	✓	✓	
		2	2	2	

Recursive sequence: (know previous terms)

$$a_k = a_{k-2} + a_{k-1}$$

$$a_{58} = a_{56} + a_{57}$$

58th

1, 2, 3, 5, 8, 13, 21, ... $a_{56} + a_{57}$

Explicit sequences (term # ↓ formula)
58th

1, 3, 5, 7, ... $2n - 1$
 $2(58) - 1$

Factorial



$$5! = 120$$

$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$\underline{21} \quad \underline{20} \quad \underline{19} \quad \dots$$

Simplify:

$$\frac{10!}{8!} \quad \frac{10 \cdot 9 \cdot \cancel{8} \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}{\cancel{8} \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}$$

90

Write the first 5 terms:

$$a_n = \frac{n^2}{(n + 1)!}$$

Summation



1, 3, 5, 7, 9, ...

Sequence

1 + 3 + 5 + 7 + 9

Series

Find the sum:

$$\sum_{k=3}^8 (3k - 2) = 87$$

3	4	5	6	7	8
7	+10	+13	+16	+19	+22

=

Series:

Finite $\sum_{k=3}^8 (3k - 2) = 87$ divergent

infinite $\sum_{x=1}^{\infty} \frac{3}{10^x} = \left(\frac{1}{3}\right)$ convergent

$$\frac{3}{10} + \frac{3}{100} + \frac{3}{1000}$$
$$.3 + .03 + .003$$

$$.3333\dots$$

